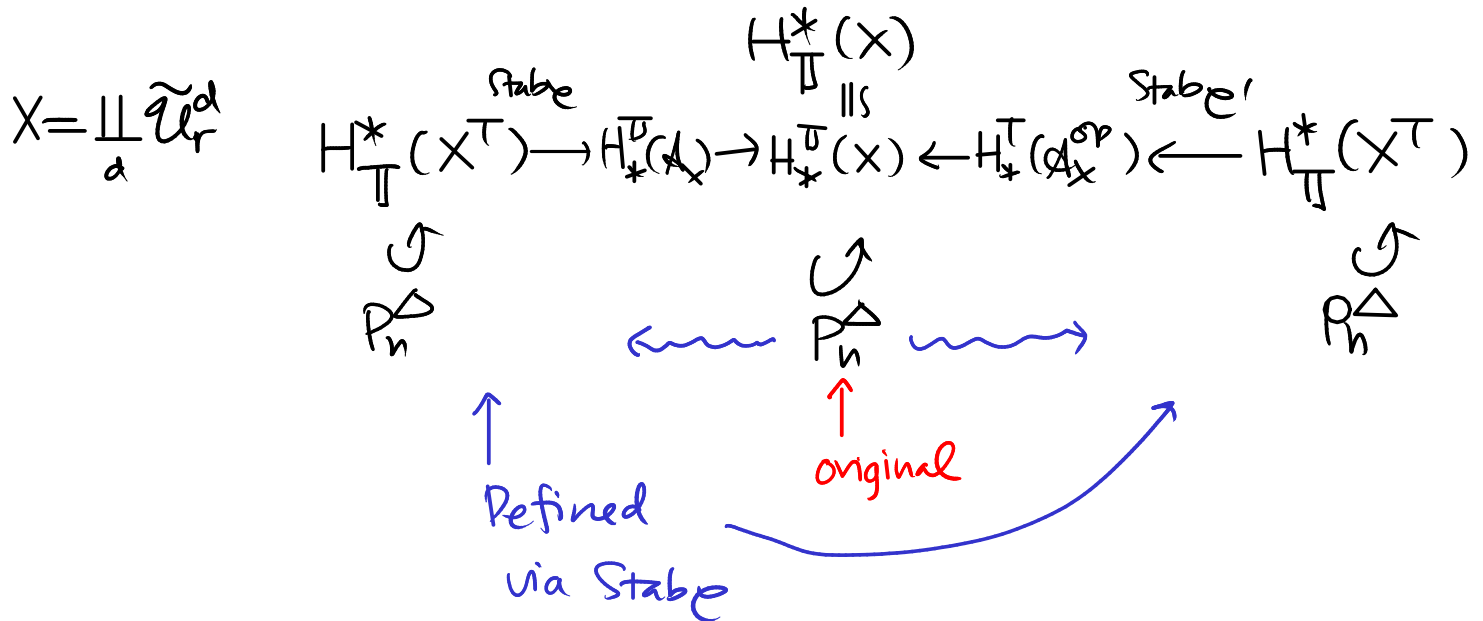


Correction  $R$ -commutes with  $P_n^\Delta = P_n^{(1)} + P_n^{(2)} + \dots$   
 o This is trivial.



In the same way,  $C_R(U)$ : Chem class of  $U$

$$\left( \mathbb{E} \Big|_{(E, \varphi)} = H^1(E(-2)) \right) \begin{array}{l} \text{tautological} \\ \text{bundle} \end{array} \uparrow \\
 \text{or } \mu^1(0) \times_{\mathbb{Q}(U)} U \text{ in given var.}$$

commutes with  $R$

o Addendum

$$Z_{\mathcal{G}} = \alpha_X \times_{X_0^T} X^T$$

$$Z_{\mathcal{G}'} = X^T \times_{X_0^T} \mathbb{R}_X \parallel \alpha_X \text{ for } -\mathcal{C} \text{ or } \lim_{t \rightarrow \infty}$$

$$(Z = X \times_{X_0} X)$$

$$H_{\mathbb{T}}^*(Z_{\mathcal{G}'}) \otimes H_{\mathbb{T}}^*(Z) \otimes H_{\mathbb{T}}^*(Z_{\mathcal{G}}) \longrightarrow H_{\mathbb{T}}^*(Z^T)$$

## § Key points of the proof of $\mathcal{R}[MO]$

$R =$  reflection operator

1. How to compute  $R$ ?

2. Why Virasoro alg. appears?

2.  $\leftarrow \mathcal{R}[Lelin '99]$

For  $Hilb^d$ -case

$$[a(\nu), P_n^\Delta] = \frac{n}{2} \sum : \underbrace{P_m P_{n-m}} : - \frac{n(|n|-1)(\epsilon_1 + \epsilon_2)}{2} P_n$$

↳ Virasoro (ordinary)

[MO]: higher rank generalization

(The strange  $|n|$  term does not affect the argument)  
→ come back later

1°  $[R, P_n^\Delta] = 0 \Rightarrow$  Enough to compute  $(R(\text{vac} \otimes \text{Fock}), \text{vac} \otimes \text{Fock})$

$\text{Stab}_{-e}^{-1} \circ \text{Stab}_e \uparrow \text{Hilb}^0 \times \text{Hilb}^d$

$\circ \text{Stab}_{-e}^{-1} = (-1)^{\frac{1}{2} \text{codim } X^T} \mathcal{L}^t$       transpose

☹ Recall  $\mathcal{L} \subset \mathcal{A}_X \times_{X_0^T} X^T \subset X \times X^T$

$\begin{matrix} p_1 \swarrow & & \searrow p_2 \\ \mathcal{A}_X & & X^T \end{matrix}$        $p_{1*}(p_2^*(\cdot) \cap \mathcal{L})$

$\mathcal{L}^t = p_{2*}(p_1^*(\cdot) \cap \mathcal{L})$        $p_2$ : not proper or use app. sol

(equivariant int, dt.)

$\text{Stab}_e^t \circ \text{Stab}_{-e} = p_{13*}(p_{12}^* \mathcal{L}^t \cap p_{23}^* \mathcal{L}) = \Delta_{X^T}$

$\begin{matrix} \text{X}^T \times \mathcal{A}_X & \cap & \mathcal{A}_X \times \text{X}^T \\ \text{X}^T \times \mathcal{A}_X & & \mathcal{A}_X \times \text{X}^T \\ & \text{"op"} & \end{matrix}$

$\begin{matrix} \text{X}^T \times X \times X^T \\ p_{12} \swarrow & & \searrow p_{23} \\ \text{X}^T \times X & & X \times X^T \end{matrix}$

$\therefore (R(\text{vac} \otimes F), \text{vac} \otimes F) = (\text{Stab}_e(\text{vac} \otimes F), \text{Stab}_{-e}(\text{vac} \otimes F))$

$\text{Stab}_{-e}^{-1} \circ \text{Stab}_e$

" / "

$\text{Stab}_e^t$

$(\widetilde{\mathcal{U}}_2^d)^{\mathbb{C}^*} = \coprod_{d=d_1+d_2} \text{Hilb}^{d_1} \times \text{Hilb}^{d_2}$

Lemma  $\text{Hilb}^0 \times \text{Hilb}^d$  is the minimum element.

$$\Rightarrow \text{Stab}_e(\text{vac} \otimes F) = \text{Leaf} \text{ for } \text{Hilb}^0 \times \text{Hilb}^d \quad (\text{like } \times \begin{pmatrix} * \\ 0 \end{pmatrix})$$

$$\begin{aligned} T_{(E, \varphi)}^{\mathcal{U}^d} &= \text{Ext}^1(E, E(-l\omega)) & E &= I_1 \oplus I_2 \\ &= \bigoplus_{\alpha, \beta=1}^2 \text{Ext}^1(I_\alpha, I_\beta(-l\omega)) \oplus \\ & & (\alpha, \beta) &= (1, 1) \quad T_{I_1} \text{Hilb}^{d_1} \\ & & & (2, 2) \quad T_{I_2} \text{Hilb}^{d_2} \end{aligned}$$

Normal bde = mixed term

$$\begin{aligned} \Rightarrow \text{Leaf} &= \text{Ext}^1(I_2, I_1(-l\omega)) \\ (\text{Leaf}^- &= \text{Ext}^1(I_1, I_2(-l\omega))) \end{aligned}$$

$$\therefore (\text{Stab}_e(\text{vac} \otimes F), \text{Stab}_e(\text{vac} \otimes F))$$

$$= \int_{\text{Hilb}^0 \times \text{Hilb}^d} \frac{e(\text{Leaf}^-)^2}{e(N)} = \int_{\text{Hilb}^0 \times \text{Hilb}^d} \frac{e(\text{Leaf}^-) e(\text{Leaf}^-)}{e(\text{Leaf}^+) e(\text{Leaf}^-)}$$

$\text{Leaf}^- = (\text{Leaf})^\vee$  by symplectic form

↑ invariant under  $T$   
but not under  $\Pi$

$$\text{Leaf}^- = \text{Leaf} \otimes e^{\varepsilon_1 + \varepsilon_2} \quad \text{"}t_1 t_2 \quad t^* \omega = t_1 t_2 \omega$$

$$= \int_{\text{Hilb}^d} \frac{e(\mathcal{U} \otimes e^{\varepsilon_1 + \varepsilon_2})}{e(\mathcal{U})} \rightarrow \text{Chem classes of } \mathcal{U}.$$

# § Intersection cohomology of Uhlenbeck spaces

[Choss-Ginzburg, Chap 8]

$D^b(X) = D_{\text{cons}}^b(X)$  : derived category of constructible sheaves on  $X$

$f: X \rightarrow Y$        $f_!, f_*, f^*, f^!, \otimes^L, R\text{Hom}, \mathbb{D}_X$  : Verdier duality

$\pi: X \rightarrow X_0$        $\leftarrow$  smooth      proper  
decomposition thm [BBDG]

$$\pi_* \mathbb{C}_X \cong \bigoplus IC(X_\alpha, L_\alpha)[d_\alpha]^{\oplus m_\alpha} \quad (\text{semisimple cpx})$$

$\uparrow$   
 $\pi^! \mathbb{C}_X$

Def. (Boris-MacPherson)

$\pi: X \rightarrow X_0$  semismall  $\Leftrightarrow \exists X_0 = \coprod X_\alpha$  stratification  
s.t.  $f|_{f^{-1}(x_\alpha)}$  top. fibration  
&  $\dim f^{-1}(x_\alpha) \leq \frac{1}{2} \text{codim } X_\alpha$

$$\Rightarrow \pi_* \mathbb{C}_X[\dim X] \cong \bigoplus_{\alpha} IC(X_\alpha, \mathcal{L}) \otimes H_{\text{top}}(\pi^{-1}(x))_X$$

$\mathbb{C}_X \cong$        $\uparrow$  local system       $\uparrow$   $X_\alpha$

without shift

loc. system over  $X_\alpha$        $H_{\text{top}}(\pi^{-1}(x)) = \bigoplus H_{\text{top}}(\pi^{-1}(x))_X \otimes \mathcal{L}$

Fact [Kaledin] symplectic resolution is semismall

$\pi^T, \pi$  : semismall       $\therefore \frac{\pi_*^T \mathbb{C}_{X^T}}{\pi_* \mathbb{C}_X}$  : s.s. Perv. sheaf (without shift)

Rem.  $\pi: \tilde{\mathcal{U}}_r^d \rightarrow \mathcal{U}_G^d$   $(E, \varphi) \mapsto (E^{uv}, \varphi), \text{Supp } \frac{E^w}{E}$

$\parallel$

$2dr$   $\coprod_{|\lambda|+d'=d} \text{Bun}_G^{d'} \times S_\lambda \mathbb{A}^2$   $\uparrow 2l(\lambda) + 2d'r$

$\mathcal{M}_\theta(V, W)$ : semi-simple rep codim =  $2(d-d')r - 2l(\lambda)$   
=  $2|\lambda|r - 2l(\lambda)$

simple  $\Leftrightarrow$  stable & costable ( $\Leftrightarrow$  transpose is stable)  
 $\circ W=0 \cdot [B_1, B_2]=0$

$\Rightarrow \dim V = 1$   $B_1, B_2$  : scalar

Fact  $\dim \pi^{-1}(x) = \frac{1}{2} \text{codim } X_\alpha \quad x \in X_\alpha$   
 $\pi^{-1}(x)$ : irreducible

This is used in Baranovsky's result.

$[\pi^{-1}(d, 0)] \quad d \cdot 0 \in S_{(d)}^d \mathbb{A}^2 \leftarrow \text{most singular stratum}$

$\left( \begin{array}{l} H_{2(dr-1)}^{\text{ord}, \mathbb{T}}(\tilde{\mathcal{U}}_r^d) \stackrel{\text{PD}}{\cong} H_{\mathbb{T}, c}^*(\tilde{\mathcal{U}}_r^d) \\ P_{-d}^\Delta([0]) \cdot \langle \text{vac} \rangle \xrightarrow{\cong} 1 \in H_{\mathbb{T}}^0(\tilde{\mathcal{U}}_r^0 = \text{pt}) \end{array} \right. \quad \text{codim} = 2dr - 2$

$\therefore \pi_* \mathcal{C}_{\tilde{\mathcal{U}}_r^d} = \bigoplus_{|\lambda|+d \leq d} \text{IC}(\text{Bun}_G^{d'} \times S_\lambda \mathbb{A}^2) \otimes H_{[0]}(\pi^{-1}(d, 0))$

$\Rightarrow H_{\mathbb{T}}^*(\tilde{\mathcal{U}}_r^d) = \text{IH}_{\mathbb{T}}^*(\mathcal{U}_G^d) \oplus \text{smaller}$   
 $\uparrow$   
 $d'=d$

moreover smaller pieces can be understood recursively.

$$\exists : \mathcal{U}_G^{d'} \times \underbrace{A^2 \times \dots \times A^2}_{\mathcal{L}(\lambda)} / \text{Stab}(\lambda) \longrightarrow \overline{\text{Bun}_G^{d'} \times S_\lambda A^2} \subset \mathcal{U}_G^d$$

extend to  
a finite  
morphism

$$\text{Bun}_G^{d'} \times S_\lambda A^2 \xrightarrow{\cong} \cup$$

$$\text{IC}(\text{Bun}_G^{d'} \times S_\lambda A^2) = \exists_* \left( \text{IC}(\text{Bun}_G^{d'}) \boxtimes \mathbb{C}_{A^2 \times \dots \times A^2 / \text{Stab}(\lambda)} \right)$$

$$\Rightarrow \bigoplus_d H_{\mathbb{T}}^*(\tilde{\mathcal{U}}_r^d) \cong \bigoplus_d \text{IH}_{\mathbb{T}}^*(\mathcal{U}_G^d) \otimes \left( \text{Fock space for } P_n^\Delta(1) \right)$$

On  $\text{IH}_{\mathbb{T}}^*(\mathcal{U}_G^d)$ , the mysterious part disappear:

$$[a(\mathcal{U}), P_n^\Delta] \Big|_{\text{IH}_{\mathbb{T}}^*(\mathcal{U}_{\text{SL}_2}^d)} = n L_n \quad (\text{FF})$$

$r=2 \Rightarrow$  We get enough big algebra!

$$\bigoplus_d H_{\mathbb{T},c}^*(\tilde{\mathcal{U}}_2^d) \cong \bigoplus_d \text{IH}_{\mathbb{T},c}^*(\mathcal{U}_G^d) \otimes \left( \text{Fock space for } P_n^\Delta([0]) \right)$$

$\downarrow \text{Virasoro}$   
 $\downarrow \text{Feigin-Fuchs}$   
 $\text{Heis}(P_n^-)$

$\uparrow \hat{\mathfrak{sl}}_1 = \text{Heis.}$   
 $\downarrow$

$$\bigoplus_d H_{\mathbb{T}}^*((\tilde{\mathcal{U}}_2^d)^T) \cong \left( \text{Fock for } P_n^- \right) \otimes \left( \text{Fock for } P_n^+ \right)$$

- Sketch theoretical analysis of convolution algebras

$$H_*(X \times_{X_0} X) \cong \text{Ext}^*(\pi_* \mathbb{C}_X, \pi_* \mathbb{C}_X)$$

$\text{s.s.} \quad \parallel \text{ decomposition thm}$   
 $\oplus \text{IC}(X_\alpha, L_\alpha)[d_\alpha]^{\oplus m_\alpha}$

(sketch of the proof)

$$\begin{array}{ccc}
 X \times_{X_0} X & \xrightarrow{i} & X \times X \\
 \pi \times \pi \downarrow & & \downarrow \pi \times \pi \\
 \Delta X_0 & \xrightarrow{\Delta} & X_0 \times X_0
 \end{array}$$

$$\begin{aligned}
 H_*(X \times_{X_0} X) &= H^*(X \times_{X_0} X, i^! \mathbb{C}_{X \times X}) \\
 &= H^*(\Delta X_0, \underbrace{(\pi \times \pi)_* i^! \mathbb{C}_{X \times X}}_{\Delta^! (\pi \times \pi)_* \mathbb{C}_X^\vee[\ ]}) \\
 &= H^*(\Delta X_0, \text{Hom}(\pi_* \mathbb{C}_X, \pi_* \mathbb{C}_X)) \\
 &= \text{Ext}^*(\pi_* \mathbb{C}_X, \pi_* \mathbb{C}_X)
 \end{aligned}$$

- If  $\pi$ : semi-small

$$\Rightarrow H_{[0]}(X \times_{X_0} X) \cong \bigoplus \text{End} (H_{\text{top}}(\pi^{-1}(x))_x)$$

$\uparrow$  s.s. alg.



Our  $\Sigma_g = \mathcal{A}_X \times_{X_0^T} X^T$  is a variant of  $X \times_{X_0} X$

$$\begin{array}{ccccc}
 X^T & & \mathcal{A}_X = \pi^{-1}(\mathcal{A}_{X_0}) & \rightarrow & X \\
 \pi^T \downarrow & & \downarrow & & \downarrow \pi \\
 X_0^T & \xleftarrow{p} & \mathcal{A}_{X_0} & \xrightarrow{j} & X_0
 \end{array}$$

Prop [1207.0529]

$$H_*(\Sigma_g) \cong \text{Ext}^*(\pi_*^T \mathbb{C}_{X^T}, p_* j^! \pi_* \mathbb{C}_X)$$

proof) exercise

Want  $H_{[0]}(\Sigma_g) \cong \text{Hom}(\pi_*^T \mathbb{C}_{X^T}, p_* j^! \pi_* \mathbb{C}_X)$

$$\Rightarrow \mathcal{L} : \pi_*^T \mathbb{C}_{X^T} \rightarrow p_* j^! \pi_* \mathbb{C}_X \quad (\cong)$$

$$\text{or } p_! j^* \pi_! \mathbb{C}_X \rightarrow \pi_! \mathbb{C}_{X^T}$$

$$\rightsquigarrow \text{End}(\pi_! \mathbb{C}_X) \rightarrow \text{End}(\pi_! \mathbb{C}_{X^T})$$

$\therefore$  coproduct

But why  $p_* j^!$  (s.s. Perv.) still perverse?

- 2 parts
- direct sum of shifts of simple perverse sheaves
    - ← decomposition thm ↔ theory of wts
  - shift does not appear
    - ← semi-smallness ↔ dimension estimates
- Braden

Mirkovic-Vilonen

(= Varagnolo-Vasserot for symplectic resolution)

Exercise Use  $\mathcal{K}$  (deformation) to give another proof.

## § hyperbolic restriction

$X \hookrightarrow T$   
Choose  $\mathcal{C}$

$$X^T \xleftarrow[p_i]{P} \mathcal{A}_X \xrightarrow{j} X$$

disjoint union according to conn. comps of  $X^T$

(We apply to  $X$ : affine)

Define  $\Phi = p_* j^! : D_{\text{const}}^b(X) \rightarrow D_{\text{const}}^b(Y)$

Th [Braden]

$$p_* j^! \cong p_! j^*$$

on weakly  $T$ -equivariant objects

( $\mu: T \times X \rightarrow X$  action

$$\mu^* \mathcal{F} \cong L \boxtimes \mathcal{F} \quad \text{loc. const. } L)$$

where

$$X^T \xleftarrow[p_i]{P} \mathcal{R}_X \xrightarrow{j^-} X$$

$\parallel$   
 $\mathcal{A}_X^{\text{op}}$

opposite chamber

Cor.  $\Phi$  preserves the purity

So  $\text{IC}(X)$  is sent to a direct sum of

shifts of IC sheaves on  $X^T$

⊖  $p_*, j^!$  increase weights

$p_!, j^*$  decrease " //

$\mathbb{R}$  [MV]

Assume  $X_0^T = \{x\}$  and  $X$  has a stratification  $X = \coprod X_\alpha$   
 s.t.  $IC(X, \mathcal{L})|_{X_\alpha}$  : loc. constant  
 $\dim \mathcal{A}_X \cap X_\alpha \leq \frac{1}{2} \dim X_\alpha$   
 $\dim \mathcal{Q}_X \cap X_\alpha \leq \frac{1}{2} \dim X_\alpha$  "  $\mathcal{A}_X \xrightarrow{j} X$   
 $\downarrow p$   
 $\{x\} = X^T$

$\Rightarrow p_* j^! (IC(X, \mathcal{L}))$  is perverse

(proof)

$$H^k(p_* j^* i_{X_\alpha}^* IC(X, \mathcal{L}))$$

$$\mathcal{A}_X \cap X_\alpha \xrightarrow{j} X_\alpha \xrightarrow{i} X$$

$$= H_c^k(\mathcal{A}_X \cap X_\alpha, j^* i_{X_\alpha}^* IC(X, \mathcal{L})) \quad \{x\}$$

$$x \in X_\alpha^T \quad H_c^k(\mathcal{A}_X \cap X_\alpha) = 0 \text{ if } k > 2 \dim \mathcal{A}_X \cap X_\alpha$$

$$i_{X_\alpha}^* IC(X, \mathcal{L}) \text{ has degree } \leq - \dim X_\alpha$$

$$H_c^k(p^{-1}(x), (j \circ \tilde{i})^* i_{X_\alpha}^* IC(X, \mathcal{L})) \neq 0 \Rightarrow k \leq 2 \dim \mathcal{A}_X \cap X_\alpha - \dim X_\alpha$$

$\leq 0$   
Assump.

dual  $\Rightarrow k \geq 0$  //

0 affine Grassmann

$G$ : reductive group /  $\mathbb{C}$   $> T$ : max. torus

$$\mathcal{O} = \mathbb{C}[[z]] \subset K = \mathbb{C}((z))$$

$$G(\mathcal{O}) \subset G(K)$$

$$\text{Gr}_G := G(K)/G(\mathcal{O}) \quad : \text{affine Grassmann}$$

$\lambda \in \text{Hom}(\mathbb{C}^*, T)$   $\mapsto$  point in  $\text{Gr}_G$   
coweight

$$\text{Gr}_G = \coprod_{\lambda: \text{dominant coweight}} \text{Gr}_G^\lambda = \bigcup G_{\mathcal{O}} \cdot \lambda$$

$\text{Perv}_{G_{\mathcal{O}}}(\text{Gr}_G)$ : category of  $G_{\mathcal{O}}$ -equivariant perverse sheaves on  $\text{Gr}_G$

semisimple,  $\{ \text{simple } \zeta = \text{IC}(\overline{\text{Gr}_G^\lambda}) \}$

Th (Lusztig, Drinfeld, Ginzburg, Mirkovic-Vilonen)

$$\text{Perv}_{G_{\mathcal{O}}}(\text{Gr}_G) \cong \text{Rep}(G^\vee)$$

$G^\vee$ : Langlands dual

$$\text{IC}(\overline{\text{Gr}_G^\lambda}) \leftrightarrow \mathcal{V}(\lambda) \quad : \text{irr. rep with h.w.} = \lambda$$

convolution  $\longleftrightarrow \otimes$

[MTV]: Weight space decomp.  $\mathcal{V}(\lambda) = \bigoplus_{\mu} \mathcal{V}_{\mu}(\lambda)$  in the  $\text{Gr}_G$ -side.

Note  $T \curvearrowright \text{Gr}_G$  fixed pts =  $\text{Gr}_T = \text{Hom}(\mathbb{C}^*, T)$  discrete "Hom( $T^\vee, \mathbb{C}^*$ )"

Fix  $\rho: \mathbb{C}^* \rightarrow T$  generic coweight (positive chamber) and define  $\infty/2$ -orbits

$$S_{\mu} := \{ x \in \text{Gr}_G \mid \lim_{t \rightarrow 0} \rho(t)x = \mu \}$$

$$T_{\mu} := \{ x \in \text{Gr}_G \mid \lim_{t \rightarrow \infty} \rho(t)x = \mu \}$$

$$\begin{aligned}
 p_* j^! : \text{Perv}_{G_\Theta}(\mathcal{G}r_G) &\longrightarrow D_{\text{cont}}^b(\mathcal{G}r_T) = \bigoplus_{\mu \in \text{Hom}(\mathbb{C}^*, T)} D^b(\text{Vect}) \\
 \parallel & \\
 \bigoplus \overline{\Phi}_\mu & \quad \quad \quad \mathbb{Z}\text{-graded vect. sp.}
 \end{aligned}$$

Prop  $\overline{\Phi}_\mu(\text{Perv}_{G_\Theta}(\mathcal{G}r_G))$  is concentrated in degree  $2\langle \rho, \mu \rangle$

key of the proof: dimension estimate

$$\begin{aligned}
 \dim S_\mu \cap \overline{\mathcal{G}r^\lambda} &= \langle \rho, \mu + \lambda \rangle \\
 \dim T_\mu \cap \overline{\mathcal{G}r^\lambda} &= -\langle \rho, \lambda + \mu \rangle
 \end{aligned}$$

MV cycle : Irreducible components of  $S_\mu \cap \overline{\mathcal{G}r^\lambda}$   
 $\rightarrow$  base of  $T_\mu(\lambda)$

# § Uhlenbeck space

$G$ : semisimple, simply-connected alg. grp /  $\mathbb{C}$

$$\mathcal{U}_G^d : \text{Uhlenbeck space} \\ = \coprod_{\lambda} \text{Bun}_G^{d'} \times S_{\lambda} \mathbb{A}^2 \quad \lambda \vdash (d-d')$$

$$\mathbb{G} = G \times \mathbb{C}^* \times \mathbb{C}^* \\ \cup \\ \mathbb{T} = T \times "$$

Choose  $\rho: \mathbb{C}^* \rightarrow T$  1 PS

$$L \leftarrow P \rightarrow G$$

$$L := G^{P(\mathbb{C}^*)} \quad \text{Levi}$$

$$P := \{g \in G \mid \lim_{t \rightarrow 0} \rho(t) g \rho(t)^{-1} \text{ exists} \} \quad \text{parabolic}$$

e.g.  $\rho(t) = \begin{bmatrix} t^{m_1} & & \\ & t^{m_2} & \\ & & \ddots \\ & & & t^{m_k} \end{bmatrix} \quad m_1 > m_2 > \dots > m_k$

$\rho$  defines a  $\mathbb{C}^*$ -action on  $\mathcal{U}_G^d$

$$\begin{array}{ccc} (\mathcal{U}_G^d)^{\mathbb{C}^*} & \xleftarrow{\rho} & \mathcal{A} \xrightarrow{\delta} \mathcal{U}_G^d \\ \Downarrow & & \Downarrow \\ \mathcal{U}_L^d & & \mathcal{U}_P^d \end{array}$$

depending only on  $L$  &  $P$

$\mathcal{U}_L^d \xleftarrow{\cong} \mathcal{U}_{[L,L]}^d$  homeo. No need to make distinction

$$\rho: \text{generic} \Rightarrow \begin{array}{ll} L = T & \text{torus} \\ P = B & \text{Borel} \end{array}$$

$$\mathcal{U}_T^d = S^d \mathbb{A}^2$$